9.2.1 NEWTON'S LAWS OF MOTION[M12, M33]

Sir Isaac Newton (1643–1727) made many observations of moving bodies and formulated a number of Laws that describe their fundamental behaviour. His Laws of motion were first published in his work Philosophiae Naturalis Principia Mathematica in 1687, and form the basis for what we know as classical mechanics (*cf.* quantum mechanics).

9.2.1.1 Force

Force is the name given to a net influence that causes a free body with mass to accelerate. Force is a vector quantity, defined as the rate of change of momentum induced in a free body by the net force acting on it, and thus has direction.

Early philosophers failed to appreciate that most ordinary objects do not move because they are in the grip of opposing but equal forces. Aristotle (384 BC–322 BC) and others believed that it was the natural state of objects on Earth to be motionless, and that they tended toward that state (eventually settling down to inertness), if left alone. This was a common experience of humans with ordinary conditions in which friction was involved, so Newton's idea that unopposed forces naturally produce constant increases in velocities, was not an obvious one. Frictional forces, acting in opposition to other kinds of forces, historically tended to hide the correct mathematical relationship between simple unopposed force and motion.

The correct behaviour for unopposed forces was first discovered by Galileo (1564–1642) in working with gravity, although it was not until Newton (1643–1727) that gravity was seen as simply producing one kind of unopposed 'force'. Newton generalised the behaviour of constant acceleration, or constant momentum gain, to forces other than gravity. He asserted in his second law of motion that this behaviour of constant momentum increase was characteristic of all forces—including the "forces" of ordinary experience, such as tension or the stress produced by pushing on an object with a finger.

We encounter many different kinds of forces in the world around us:

Forces that act on all objects:

Weight

The weight of an object is simply the force of gravity acting on that object due to its mass. An object's weight is directed downward, toward the centre of the gravitating body; like the earth or moon, for example. Thus, the weight force ($\mathbf{F}_g$) is given by the equation:

$$\mathbf{F}_g = m\mathbf{g}$$

where m is the mass of the object and $\mathbf{g}$ is the acceleration due to gravity.

Note that the weight of an object is not the same as its mass. While the mass of an object is constant, its weight depends on the prevailing gravitational force. On the moon, this is only one sixth that on earth, so while the mass of an object remains the same, its weight on earth is six times that on the moon.

Forces associated with solids:

Normal

The force between two solids in contact that prevents them from occupying the same space. The normal force is directed perpendicular to the surface. A "normal" in mathematics is a line perpendicular to a curve or surface; thus the name "normal force".

The normal force is always perpendicular to the contact surface, but not necessarily directly opposite the force of gravity (the weight force). The relationship between the normal force ($\mathbf{F}_n$) and the weight force ($\mathbf{F}_g$) is thus given by the equation:

$$\mathbf{F}_n = \mathbf{F}_g \cos\theta = m\mathbf{g}\cos\theta$$

where θ is the angle between the contact surface and the horizontal plane.

Friction

Friction is the force between solids in contact that resists their sliding across one another. Friction opposes any applied force.

The frictional force that acts on a stationery object (the **force of static friction**) is greater than the frictional force that acts on the same object, in the same environment, when it is moving (the **force of kinetic fiction**). The practical illustration of this relationship is the fact that an object is more resistant to initial movement and is easier to push once it is moving.

The force of friction is proportional to the normal force, but is dependent on the surfaces that are in contact—for example, it is easier to slide an object across a polished board floor than across carpet.

The relationship between these three forces is illustrated in **Figure 9.2.1.1** below, where $\mathbf{F}_f$ is the generalised frictional force (either $\mathbf{F}_s$ or $\mathbf{F}_k$) and all forces are shown acting through the centre of mass of the object.

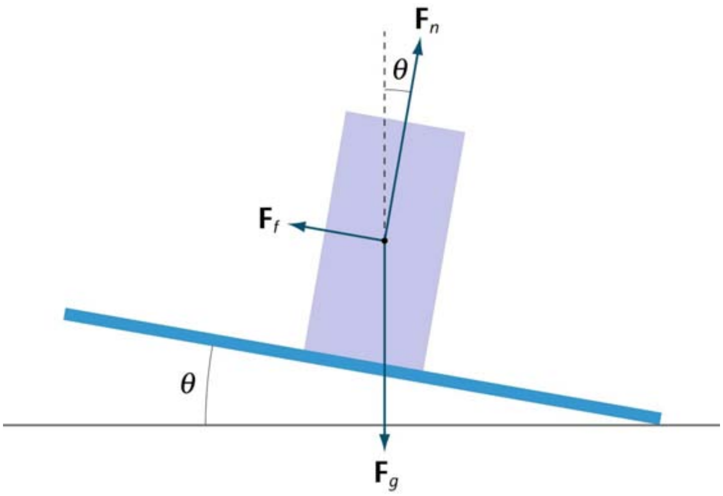


Figure 9.2.1.1 Forces acting on an object resting on an inclined plane

Tension

The force exerted by an object being pulled upon from opposite ends like a string, rope, cable, chain, etc. Tension is directed along the axis of the object. (Although normally associated with solids, liquids and gases can also be said exert tension in some circumstances.)

Elasticity

The force exerted by an object under deformation (typically tension or compression) that will return to its original shape when released like a spring or elastic band. Elasticity, like tension, is directed along an axis (although there are exceptions to this rule).

Forces associated with fluids (liquids and gases):

Buoyancy

The force exerted on an object immersed in a fluid. Buoyancy is usually directed upward (although there are exceptions to this rule).

Drag

The force that resists the motion of an object through a fluid. Drag is directed opposite the direction of motion of the object relative to the fluid.

Lift

The force that a moving fluid exerts as it flows around an object; typically a wing or wing-like structure, but also golf balls and baseballs. Lift is generally directed perpendicular to the direction of fluid flow (although there are exceptions to this rule).

Thrust

The force that a fluid exerts when expelled by a propeller, turbine, rocket, squid, clam, etc. Thrust is directed opposite the direction the fluid is expelled.

Forces associated with physical phenomena:

Electrostatic Force

The attraction or repulsion between charged bodies. Experienced in everyday life through static cling and in school as the explanation behind much of elementary chemistry.

Magnetic Force

The attraction or repulsion between magnetic bodies. Experienced in everyday life through magnets and in school as the explanation behind why a compass needle points north. (Actually, magnetism is the attraction or repulsion between charged bodies *in motion*, but this description is good enough for now. Electricity and magnetism are dealt with more thoroughly in later chapters.)

Fundamental forces.

All the forces in the universe can be explained in terms of the following four fundamental interactions:

Gravity

The interaction between objects due to their mass. Weight is the name for the force of gravity.

Electromagnetism

The interaction between objects due to their charge. All the forces discussed above except weight are electromagnetic in origin.

Strong Nuclear Interaction

The interaction between subatomic particles with *colour* (an abstract quantity that has nothing to do with human vision). This is the force that holds protons and neutrons together in the nucleus and holds quarks together in the protons and neutrons. It cannot be felt outside of the nucleus.

Weak Nuclear Interaction

The interaction between subatomic particles with *flavour* (an abstract quantity that has nothing to do with human taste). This force, which is many times weaker than the strong nuclear interaction, is involved in certain forms of radioactive decay.

Fictitious forces

These are apparent forces that objects experience in an accelerating coordinate system like an accelerating car, aeroplane, spaceship, elevator, or amusement park ride. Fictitious forces are not authentic forces in the sense that they do not arise from an external object, but rather as a consequence of trying to keep up with a changing environment.

Centrifugal Force

The force experienced by all objects in a rotating coordinate system that seems to pull them away from the centre of rotation.

Coriolis Force

The force experienced by moving objects in a rotating coordinate system that seems to deflect them at right angles to their direction of motion.

"**G Force**"

Not really a force (or even a fictitious force) but rather an apparent gravity-like acceleration experienced by objects in an accelerating coordinate system.

Pushing and **pulling** through physical contact with another moving object, friction, thrust, lift and drag, gravitational, electrostatic and magnetic interactions are all commonly experienced forces.

A force is also provided by any **strained material** in attempting to regain its shape. When elastic materials are stretched or compressed we say that **tensile strain** or **compression strain** have been produced. The **restoring force** per unit area produced by strained material is known as **stress**.

The SI derived unit of force is the **newton**, N ($kg \cdot m \cdot s^{-2}$).

9.2.1.1.1 Friction

Friction is a force that resists motion. It involves objects that are in contact with each other and is often seen as an undesirable force that reduces the efficiency of a machine.

In many instances, however, friction is very desirable. We would be unable to walk if there were no friction between the soles of our shoes and the ground. There must be friction between the tyres of a car and the road before the car can move. When we apply the brakes on the car, the friction between the brake pads and drums or discs, slows down the wheels. Friction between the tyres and the road brings the car to a stop.

In a less obvious way, friction holds screws and nails in place and it keeps plates from sliding off a table if the table is not perfectly level.

Frictional forces are commonly defined in terms of a **coefficient of friction** (μ), the ratio of the force ($\mathbf{F}_f$) resisting motion between two surfaces and the force ($\mathbf{F}_n$) holding the two surfaces together:

$$\mu = \frac{\mathbf{F}_f}{\mathbf{F}_n}$$

The coefficient of friction is generally dependent on the nature of the materials involved, and is thus greater for ‘rough’ surfaces than it is for ‘smooth’ surfaces. Furthermore, the frictional characteristics of a stationery object (defined by its **coefficient of *static* friction**) are different to the frictional characteristics of the same object when it is moving (defined by its **coefficient of *kinetic*** or ***dynamic* friction**). The coefficient of static friction depends on the area of contact between two objects, whereas the coefficient of dynamic friction does not. The latter is thus invariably smaller (*i.e.* a greater effort is required to start an object in motion that is required to keep it moving).

9.2.1.2 Newton's First Law of Motion

Newton's first law of motion deals with the motion of a body on which no net force is acting. That is, either there is no force at all acting on the body or the vector sum of all forces acting on the body is zero. The word "net" refers to the second situation. Even though a body may have many forces acting on it, these forces may act against each other. They may balance each other in such a way that the body does not change its state of motion.

If such a body is at rest, it will remain at rest. If it is in motion, it will continue in its motion in a straight line with uniform speed.

Newton's first law of motion may therefore be stated as follows:

> *Every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied to it.*

At first glance, this law seems to contradict our everyday experiences. If a car is to be kept moving with a constant velocity, the car's engine must apply a constant force to it. If the engine stops applying this force, the car comes to a stop. Only then does the car seem to obey the part of Newton's law that states that objects at rest will remain at rest unless acted upon by an unbalanced force.

Close study of a moving car shows, however, that it is the force of friction that brings the car to a stop and not the absence of the force provided by the engine. If it were possible to remove this friction, it would be reasonable to assume that the car would keep rolling without applying a constant force.

This is, in fact, what happens in space travel. Once a spaceship is beyond the pull of the earth's gravity, it continues to move with constant velocity even without the thrust of its engines.

9.2.1.2.1 Inertia

We recognize Newton’s First Law as essentially a statement of Galileo's concept of **inertia**—the property of a body that opposes any change in its state of motion. Thus, Newton’s First Law is often referred to as the *Law of Inertia* because is states that in the absence of forces, a body will preserve its state of motion.

The study of the motion of a car in the absence of friction is an example of a thought experiment since the study cannot be performed under actual conditions. It was a thought experiment of this kind that led Galileo to an understanding of inertia even before Newton described it. Galileo noticed that if a ball rolls down one incline and up a second one, the ball will reach almost the same height on the second incline as the height from which it started on the first incline.

Galileo concluded that the difference in height is caused by friction and that if friction could be eliminated the heights would be exactly alike. Then he reasoned that the ball would reach the same height no matter how shallow the slope of the second incline. Finally, if the second slope were eliminated altogether, the ball would keep rolling indefinitely with constant velocity.

This is the same idea as the one expressed in Newton's first law of motion. Inertia keeps a stationary object stationary and a moving object moving. The greater the mass (and inertia) of an object, the greater is the force required to produce a given acceleration. A pencil lying on the floor has relatively little mass and therefore little inertia. You can produce acceleration easily by kicking it with your foot. A brick has much more mass and more inertia. You can easily tell its difference from the pencil if you kick it.

Newton's first law specifies which forms of motion have a cause and which forms do not. Uniform motion in a straight line is the only motion possible for an object far removed from other objects. Non-uniform motion is always caused by the presence of some other object.

9.2.1.2.2 Equilibrium

Objects that are either at rest or moving with constant velocity are said to be in **equilibrium**. Newton’s First Law describes objects in equilibrium, whether they are at rest or moving with constant velocity. It states one condition that must be true for equilibrium: the net external force acting on a body in equilibrium must be equal to zero.

9.2.1.3 Newton's Second Law of Motion

Newton's first law of motion tells us how a body acts when there is no net applied force. Let us consider what happens when there is either a single applied force or two or more applied forces whose vector sum is non-zero. In the following discussion, we shall use *applied force* or just *force* to mean the vector sum of all forces applied to the body.

Newton's second law of motion states:

> *The relationship between an object's mass* m*, its acceleration* **a***, and the applied force* **F** *is given by the equation:*
>
> $\mathbf{F} = m\mathbf{a}$

Thus, the effect of an applied force is to cause a body to accelerate in the direction of the force. The acceleration is in direct proportion to the force and in inverse proportion to the mass of the body. This is often referred to as the **Law of Acceleration**.

If the body is at rest when the force is applied, it will begin to move in the direction of the force and will move faster and faster as long as the force continues.

If the body is moving in a straight line and a force is applied in the direction of its motion, it will increase in speed and continue to do so as long as the force continues. If the force is applied in the direction opposite to the motion, the acceleration will again be in the direction of the force, causing the body to slow down. If such a force continues long enough, the body will slow down to a stop and then begin to move with increasing speed in the opposite direction.

This is the most powerful of Newton's three Laws, because it allows quantitative calculations of dynamics: how do velocities change when forces are applied.

Notice the fundamental difference between Newton's Second Law and the dynamics of Aristotle: according to Newton, a force causes only a change in velocity (an acceleration); it does not maintain the velocity as Aristotle held. This is sometimes summarised by saying that under Newton, $\mathbf{F} = m\mathbf{a}$, but under Aristotle $\mathbf{F} = m\mathbf{v}$, where $\mathbf{v}$ is the velocity. Thus, according to Aristotle there is only a velocity if there is a force, but according to Newton an object with a certain velocity maintains that velocity unless a force acts on it to cause an acceleration (that is, a change in the velocity). As we have noted earlier in conjunction with the discussion of Galileo, Aristotle's view seems to be more in accord with common sense, but that is because of a failure to appreciate the role played by frictional forces. Once account is taken of all forces acting in a given situation it is the dynamics of Galileo and Newton, not of Aristotle, that are found to be in accord with the observations.

9.2.1.4 Newton's Law of Universal Gravitation

Newton also described the force that makes falling bodies accelerate toward the earth. In doing so, he made use of the laws of planetary motion that were developed by Johannes Kepler (1571–1630) almost a century earlier.

In Newton's account of his study of falling bodies, he states that he wondered whether the force that makes an apple fall to the ground was related to the force that keeps the planets in their orbits. If so, a single law could be used to describe the attraction between objects in the entire universe.

From Kepler's laws, Newton thus deduced the fact that:

> *Every object in the Universe attracts every other object with a force directed along the line of centres for the two objects that is proportional to the product of their masses and inversely proportional to the square of the separation between [the centres of mass of] the two objects.*

$$\mathbf{F_g} = G\frac{m_1 m_2}{r^2}$$

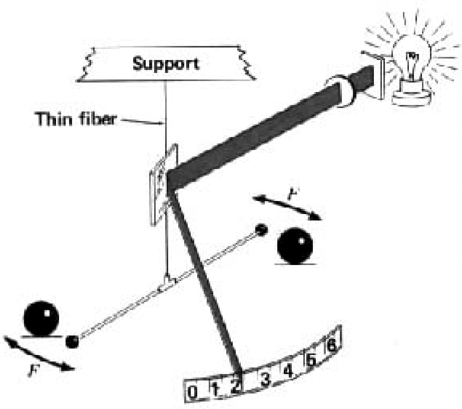
$\mathbf{F_g}$ is the gravitational force (in *newtons*)
m_1 & m_2 are the masses of the two objects (in *kilograms*)
r is the separation between the objects (in *metres*)
G is the universal gravitational constant (6.67×10^{-11} $n \cdot m^2 \cdot kg^{-2}$)

Newton called this attractive force the *force of gravitation*. Note that gravitational forces are many millions of times smaller than electrostatic forces.

9.2.1.4.1 The Cavendish Experiment

Newton was only able to confirm his theory with astronomical observations. He was unable to measure G with the laboratory instruments available at his time. The first determination of G was made in 1797 by the English scientist Henry Cavendish (1731-1810). A schematic diagram of this experiment is shown in the illustration.

Two small lead spheres are attached to the end of a lightweight rod that has a mirror attached to it. The rod is suspended by a thin quartz fibre. Two large lead spheres are placed in fixed positions near the small spheres.



The gravitational force between the fixed and movable spheres draws the movable spheres toward the fixed ones. This motion causes the suspending fibre to twist. The fibre offers a slight resistance to twisting. The resistance increases as the twisting increases, and the angle of twist is proportional to the force of gravitation between the fixed and the movable spheres.

The angle of twist could be measured directly from the rod's movement, but the sensitivity of the instrument can be increased by shining a light into the mirror. The light is reflected onto a distant scale, and small movements of the mirror result in large movements of the reflected light across the scale.

Such an arrangement is called an optical lever. Since G has a very small value, extreme care must be used to isolate the Cavendish apparatus from outside forces such as those produced by air currents and electric charges.

This experiment provided direct proof of Newton's Law of Universal Gravitation.

9.2.1.4.2 Weight and the Gravitational Force

What we commonly call weight is really the *force of gravitation* exerted on an object of a certain mass. Using the equation for Universal Gravitation and choosing Earth as one of the two masses:

$$\textbf{Weight} = \mathbf{F}_g = G\frac{Mm}{r^2} = m\mathbf{g} \qquad \left(\mathbf{g} = G\frac{M}{r^2}\right)$$

M	is the mass of the Earth
m	is the mass of the object
r	is the separation between the objects
G	is the universal gravitational constant

Thus, the weight of an object of mass m, at the surface of the Earth is obtained by multiplying the mass m by the acceleration due to gravity, **g**, at the surface of the Earth. Since the gravity on the earth may vary from place to place, however, the value of the acceleration due to gravity may also vary. The variation in the value of **g** across the earth's surface is about 0.5% due to latitude, plus a change of approximately 0.003% per 100 metres altitude. Local topography and tidal forces also can have small effects.

An approximate value for g, at a given latitude and height above sea level, may be calculated from the formula[1]:

$$g = 9.780\,318\,4\,(1 + A\sin^2 L - B\sin^2 2L) - 3.086 \times 10^{-6}\,H$$

where	A	=	0.005 302 4
	B	=	0.000 005 9
	L	=	latitude
	H	=	height in metres above sea level

The uncertainty in the value of **g** so obtained is generally less than ±5 parts in 10^5.

Nonetheless, for most purposes the acceleration due to gravity can be approximated[2] to 9.8 m·s^{-2}.

This force of attraction is called the weight-force of an object. Although we think of the direction of the weight-force as being *down*, since the earth pulls objects towards its centre, the direction of the weight-force is more correctly stated as *towards the centre of the earth*.

9.2.1.4.3 Freely Falling Bodies

The equations for accelerated motion (Section 7.2.2.6) will also apply to freely falling bodies—objects falling under the influence of gravity—if **g**, the acceleration due to gravity, is substituted for the acceleration vector **a**. By convention, the vector quantities **v** and **s** are assigned positive values if they are directed downward and negative values if

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1 http://www.npl.co.uk/mass/faqs/gravity.html

2 This is accurate to two significant digits over the entire surface of the earth, up to an altitude of 18 km

they are directed upward. The displacement, **s**, is always a vertical distance. However, the equations only apply to objects that are falling freely in a vacuum, because they do not take into consideration the resistance of air, which for objects such as leaves or feathers is large compared to their mass.

This limitation not withstanding, objects falling freely under the influence of gravity fall with the same acceleration no matter what their mass (contrary to the teachings of Aristotle), and accelerate uniformly, that is, their velocity changes by the same amount each second. An object thrown upward is uniformly decelerated by the force of gravity until it finally stops rising then, as the object falls, it is uniformly accelerated by the same force. If the effect of the atmosphere is neglected, then the time required for the object to fall is the same as the time required for it to rise.

9.2.1.5 Newton's Third Law of Motion

Newton's third law of motion may be stated as follows:

For every action there is an equal and opposite reaction.

When one body exerts a force on another, the second body exerts on the first a force of equal magnitude in the opposite direction. This is often referred to as the **Law of Interaction**. It is exemplified by what happens if we step off a boat onto the bank of a lake: as we move in the direction of the shore, the boat tends to move in the opposite direction (leaving us facedown in the water, if we're not careful!).

To further illustrate this law, consider some of the forces that are exerted when a book is resting on the top of a level table. The book exerts a downward force against the table. The table top exerts an upward force on the book. These forces are equal in magnitude and opposite in direction.

When you walk forward on a level floor your feet exert a horizontal force against the floor, and the floor pushes against your feet with a force of equal magnitude, but in the opposite direction.

In each of these situations we have two objects. In the first instance, the objects are the book and the table. In the second instance, they are the foot and the floor. Two forces are involved in each situation. In the first, they are the force of the book against the table and the force of the table against the book. In the second, they are the force of the foot against the floor and the force of the floor against the foot.

In cases such as these, one force may be called the action, while the second force may be called the reaction. Unaccompanied forces do not exist in nature. The law of reaction holds true for all objects at all times, whether they are stationary or moving. Every force is resisted by an equal and opposite force, independent of the motion of the objects involved.

You may think that motion cannot occur if action and reaction are equal. If two people pull on a lightweight wagon with equal force in opposite directions, the wagon will not move. But this is not an example of action and reaction. There are two forces, it is true, and they have the same magnitude and opposite directions, but they are both exerted on the same object. This is an example of resolution of force vectors that, in this case, yields a net zero force.

Action and reaction apply when forces are exerted on different objects. For example, the force that each one's feet exert against the ground and the equal but opposite force the ground exerts against the feet.

9.2.1.6 Momentum

9.2.1.6.1 The Nature of Momentum

More force is needed to stop a train than to stop a car, even when both are moving with the same velocity. A bullet fired from a gun has more penetrating power than a bullet thrown by hand, even though both bullets have the same mass.

The physical quantity that describes this characteristic of moving objects is called momentum. Momentum is the product of the mass of a moving object and its velocity, and since velocity is a vector quantity, so too is momentum. Mathematically momentum is defined by the equation:

$$\mathbf{p} = m\mathbf{v}$$

where **p** is the momentum;
m is the mass; and
v is the velocity of the object.

The SI units for momentum are $kg{\cdot}m{\cdot}s^{-1}$.

In the example of the car and train, the greater mass of the train gives it more momentum than the car. Consequently, a greater change of momentum is involved in stopping the train than in stopping the car. In the case of the bullets, the greater momentum of the fired bullet is due to its greater velocity; a large change of momentum takes place when the speeding bullet is stopped.

9.2.1.6.2 The Conservation of Momentum

In Figure 9.2.1.2, a boy with a mass of 40 kg and a man with a mass of 80 kg are standing on a frictionless surface. When the man pushes on the boy from the back, the boy moves forward and the man moves backward.

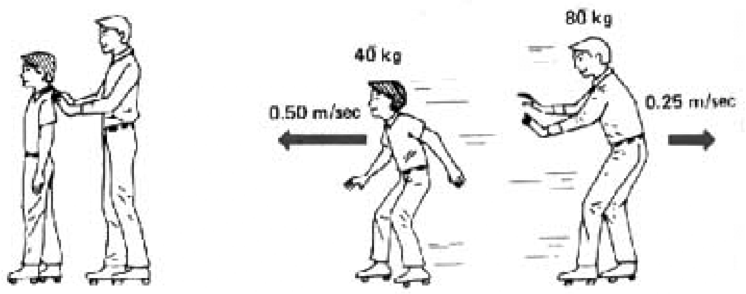

Figure 9.2.1.2 Conservation of momentum. On a frictionless surface, the momentum of the boy toward the left is equal to the momentum of the man toward the right.

The velocities with which the boy and the man move are specified by one of the most important principles of physics, the **Law of Conservation of Momentum**. This law states that:

> *When no net external forces are acting on a system of objects, the total vector momentum of the system remains constant.*

Let us apply this law to the situation in Figure 9.2.1.2. Initially the man and the boy are at rest. The system, therefore, has zero momentum. When the man and the boy move apart, the law of conservation of momentum requires that the total vector momentum remain zero. Hence the momentum of the boy in one direction must equal that of the man in the other direction.

If the boy moves with a velocity of 0.50 m·s^{-1}, the man will move with a velocity of 0.25 m/s, since the mass of the man is twice that of the boy. The momentum of the boy in one direction (40 kg × 0.50 m·s^{-1}) must equal the momentum of the man in the opposite direction (80 kg × 0.25 m·s^{-1}).

An important application of the law of conservation of momentum is the launching of a rocket. When a rocket fires, hot exhaust gases are expelled through the rocket nozzle. The gas particles have a momentum equal to the mass of the particles multiplied by their exhaust velocity. Momentum equal in magnitude is therefore imparted to the rocket in the opposite direction. Newton's Third Law of Motion is a special case of the Law of Conservation of Momentum.

9.2.1.6.3 Collisions

The Law of Conservation of Momentum is very helpful in studying the motions of colliding objects. Collisions can take place in various ways, and we shall see how the momentum conservation principles apply in several such cases.

Elastic Collisions

An **elastic** collision occurs when colliding objects rebound from each other without a loss of kinetic energy. The only perfectly elastic collisions, however, occur between atomic and subatomic particles. The situation can, nonetheless, be approximated by the use of hard steel balls or springs on an air track.

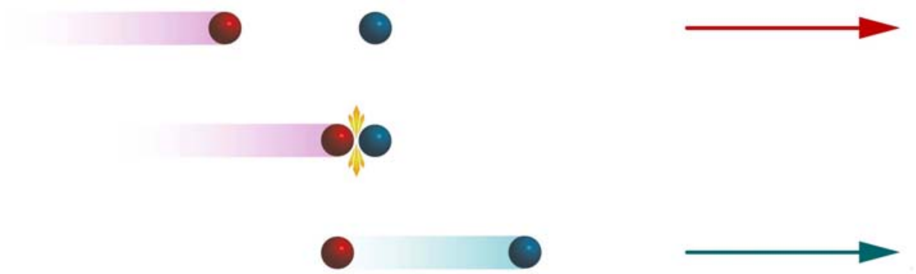

Figure 9.2.1.3 The collision between hard steel balls, approximating an elastic collision in one dimension

The Law also holds for collisions in two dimensions, that is, when the colliding objects meet at an angle other than head-on. In Figure 9.2.1.4 below, the red ball collides at an angle with the blue ball, which is initially at rest. As illustrated, vector diagrams can be used to represent the momenta of the balls before and after collision.

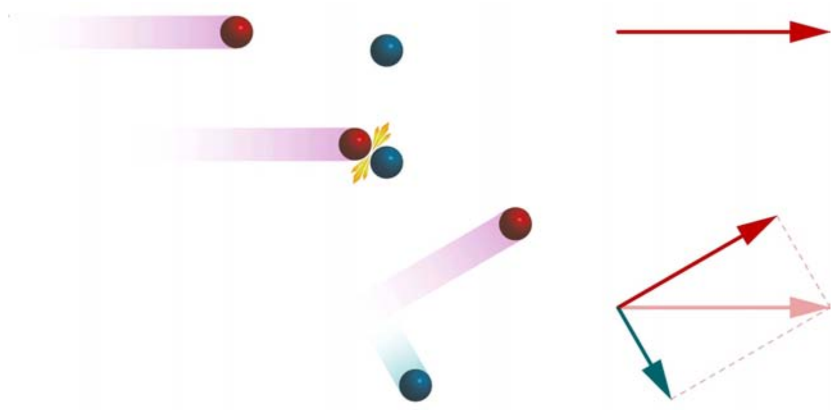

Figure 9.2.1.4 The collision between hard steel balls, approximating an elastic collision in two dimensions

So it follows from the laws of conservation of energy and momentum that, when a moving ball strikes a stationary ball of equal mass other than head-on in an elastic collision, the two balls move away from each other at right angles. The conservation of momentum also holds for collisions involving more than two bodies and in three-dimensional situations.

Inelastic Collisions

When two objects collide and stick together, so that they travel together after impact, the collision is said to be **inelastic**. Consider the example illustrated in Figure 9.2.1.5. Two carts of equal mass approach each other with velocities of equal magnitude. A lump of putty (or velcro) is attached to the front of each cart so that the two carts will stick together after the impact—an inelastic collision. Since the carts are travelling along the same straight line, it is also an example of a collision in one dimension.

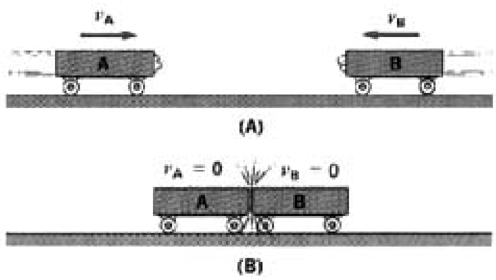

Figure 9.2.1.5 An inelastic collision. The carts have equal masses and approach each other with velocities of equal magnitude along the same straight line. The total momentum of the system is the same before and after the collision.

The momentum of cart **A** is $m_A\mathbf{v}_A$. It is equal in magnitude to the momentum of cart **B**, $m_B\mathbf{v}_B$. However, the direction of $\mathbf{v}_A$ is opposite to the direction of $\mathbf{v}_B$, so

$$\mathbf{v}_A = -\mathbf{v}_B$$

Consequently,

$$m_A\mathbf{v}_A = -m_B\mathbf{v}_B$$

and

$$m_A\mathbf{v}_A - m_B\mathbf{v}_B = 0$$

This means that the total vector momentum of the system of two moving carts is zero. (We assume that the system is isolated, that is, there are no net external forces acting on it.)

In actual collision studies, the external force of friction is usually minimised by using rolling carts or air tracks.

After the carts collide, they both come to rest. The cart velocities, $\mathbf{v}_A$ and $\mathbf{v}_B$, are now both zero; the sum of the momenta of the two carts is zero, just as it was when the carts were in motion in opposite directions.

Thus the total vector momentum of the system is unchanged by the collision. If one of the carts has a greater mass than the other, although its velocity is still of equal magnitude but opposite sign, the outcome of the collision is different. After impact, the combined carts will move in the direction of the cart with the larger mass. The velocity of the combined carts will be such that the total momentum of the system remains unchanged.

For example, suppose one cart has twice the mass of the other

$$m_A = 2m_B$$

and

$\mathbf{v}_A = -\mathbf{v}_B$ as before,

then,

$$m_A\mathbf{v}_A + m_B\mathbf{v}_B \neq 0$$

Since

$$m_B = \tfrac{1}{2}m_A$$

then by substitution,

$$m_A\mathbf{v}_A + m_B\mathbf{v}_B = m_A\mathbf{v}_A - \tfrac{1}{2}m_A\mathbf{v}_A = \tfrac{1}{2}m_A\mathbf{v}_A$$

This means that the total momentum of the system, before and after the collision, is

$$\mathbf{p}_t = \tfrac{1}{2}m_A\mathbf{v}_A$$

The combined carts will move with this momentum in the direction of the original velocity of cart **A**. The velocity after the collision can be found by dividing the total momentum by the total mass:

$$\mathbf{v}_t = \frac{\tfrac{1}{2}m_A\mathbf{v}_A}{m_A + m_B} = \frac{\tfrac{1}{2}m_A\mathbf{v}_A}{m_A + \tfrac{1}{2}m_A} = \frac{\tfrac{1}{2}m_A\mathbf{v}_A}{\tfrac{3}{2}m_A} = \tfrac{1}{3}\mathbf{v}_A$$

When carts of equal and opposite momenta collide inelastically, they come to rest. Before the collision they have **kinetic energy**; after the collision, they do not. This is typical of all inelastic and partially elastic collisions.

Much or all of the kinetic energy that the moving objects have before collision is converted into heat or some other form of energy. If all these forms of energy are taken into account, the law of conservation of energy holds true for inelastic collisions. But kinetic energy alone is not conserved.

References

Holt Physics, Serway, R.A. and Faughn, J.S. (Holt, Rinehart and Winston, 2000) [ISBN 0-03-056544-8] Ch. 4, Forces and the Laws of Motion; Ch. 6, Momentum and Collisions; Ch. 7, Rotational Motion and the Law of Gravity.

Work directly from text, with exercises:

4 Forces and the Laws of Motion

4.1 Changes in motion

4.2 Newton's first law

4.3 Newton's second an third laws

4.4 Everyday forces

6 Momentum and Collisions

6.1 Momentum and Impulse

6.2 Conservation of momentum

6.3 Elastic and inelastic collisions

Conceptual Physics Fundamentals, Hewitt, P.G. (Pearson Addison Wesley, 2008) [ISBN 0-321-50136-5] Ch. 4, Newton's Laws of Motion; Ch. 5, Momentum and Energy; Ch. 6, Gravity, Projectiles and Satellites.